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LETTER TO THE EDITOR

Spontaneous breaking of translational invariance in one-dimensional stationary states on a ring

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Abstract. We consider a model in which positive and negative particles diffuse in an asymmetric, CP-invariant way on a ring. The positive particles hop clockwise, the negative counter-clockwise and oppositely-charged adjacent particles may swap positions. Monte Carlo simulations and analytic calculations suggest that the model has three phases; a 'pure' phase in which one has three pinned blocks of only positive, negative particles and vacancies, and in which translational invariance is spontaneously broken, a 'mixed' phase with a non-vanishing current in which the three blocks are positive, negative and neutral, and a disordered phase without blocks.

Even if obtained from a master equation with local dynamics, stationary states are not necessarily given by Gibbs ensembles. In cases where one has detailed balance, the Hamiltonian may have long-range interactions. For these reasons, stationary states can exhibit phase transitions even in one dimension. Spontaneous CP-symmetry breaking was seen in open chains, in the two-state partially asymmetric exclusion model [1–3] (in which C interchanges particles and vacancies) as well as in the three-state model with positive particles, negative particles and vacancies [4] (in which C interchanges positive and negative particles). In both models, macroscopic structures in the form of shocks exhibiting phase separation appear at the phase transition point between the disordered and the broken phases [1, 5, 6]. In the present letter we describe a model defined on a ring which shows spontaneous breaking of translational invariance and phase separation, as well as a new type of macroscopic structure. We will describe here only some results, a complete presentation being given elsewhere [7].

We are considering a ring with L sites numbered $k = 0, 1, \ldots, L-1$. On each site k one may have a positive particle $(+)_k$, a negative particle $(-)_k$ or a vacancy $(0)_k$. The time evolution of the system is given by the following rates:

$$\begin{aligned}
&(+)_k(0)_{k+1} \to (0)_k(+)_{k+1} && \text{rate } 1\\
&(0)_k(-)_{k+1} \to (-)_k(0)_{k+1} && \text{rate } 1\\
&(-)_k(+)_{k+1} \to (+)_k(-)_{k+1} && \text{rate } 1\\
&(+)_k(-)_{k+1} \to (-)_k(+)_{k+1} && \text{rate } q. \qquad (1)
\end{aligned}$$

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The other rates are zero. These processes conserve the numbers of positive and negative particles and are CP-invariant. We shall consider only configurations in which the average densities of positive and negative particles, denoted by p and m, respectively, are equal.

We mention that for the model we are considering the stationary state can be computed analytically using the matrix-product approach [3] for any value of q [8]. The parameter q determines the physics of the problem.

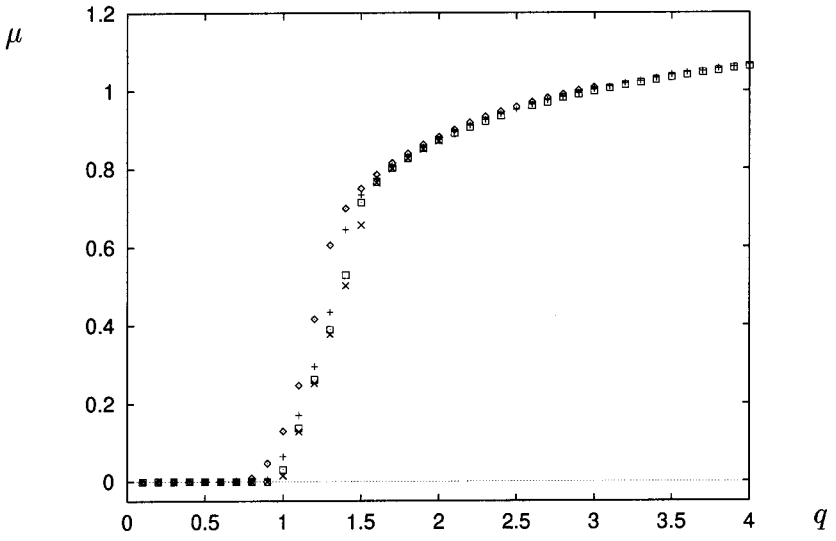

Figure 1. The mobility as a function of q for $p = m = 0.2$ and $L = 100(\diamond)$, $200(+)$, $400(\square)$, $800(\times)$. The errors in this and the following figures are less than the size of the symbols.

In figure 1 we show, based on results of Monte Carlo simulations, the mobility μ (defined as the ratio of current to density) as a function of q for $p = m = 0.2$ and different values of L. One can distinguish three regions. For $q < 1$ the mobility converges exponentially to zero. This region will be named, for reasons which will become apparent soon, as the 'pure' phase. It is separated by a critical point, $q = 1$, from the 'mixed' phase. At $q = 1$, the current vanishes algebraically with L with an exponent 1.05 ± 0.05. In the 'mixed' phase, $1 < q < 1.4$, the mobility is non-zero and it approaches its limiting value algebraically. The 'mixed' phase is separated by a second critical point q_c from the disordered phase. In the disordered phase, the mobility converges exponentially. In figure 2 we show the mobility for three densities at $L = 200$. One notices a remarkable fact. For $q = 3$ the mobilities are all equal to 1 independent of the density or, although not seen in the figure, the lattice size. The explanation is simple. Using the matrix-product approach which uses representations of a quadratic algebra [8], one can show that for $q = 3$ one obtains a one-dimensional representation of the quadratic algebra. One can then show that the mobility is equal to 1 for any density and lattice size. The same approach can be used for $q = 4$ where one has a two-dimensional representation of the quadratic algebra. This allows one to compute the mobility and check that the values obtained by Monte Carlo simulations are correct. We have also computed the correlation length and shown that it is finite. This shows that for $q = 4$ one is in the disordered phase for any density. There are no other finite-dimensional representations of the quadratic algebra for other values of q. However, for other exact results see [7].

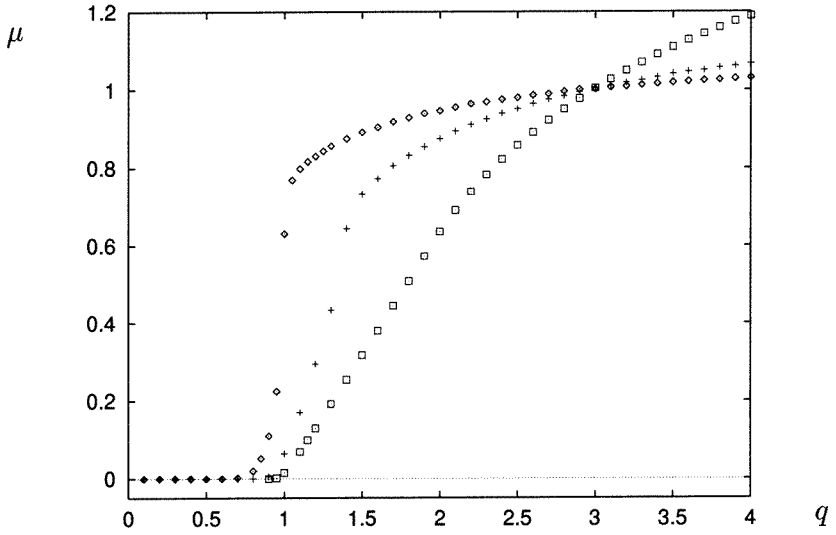

Figure 2. The mobility as a function of q for $p = m = 0.1(\diamond)$, $0.2(+)$, $0.4(\square)$ and $L = 200$.

Repeating the same simulations for various lattice sizes, we have seen that $q = 1$ is the limit of the 'pure' phase for any density but that the convergence to zero is slower for small densities. The second critical point q_c increases with the density. We found $q_c = 1.4 \pm 0.1$ for $p = m = 0.2$ and $q_c = 1.9 \pm 0.1$ for $p = m = 0.4$.

We now proceed to clarify the structure of the three phases. We start with the 'pure' phase. At $q = 0$, a single vacancy is sufficient to break the translational invariance of the system. At a finite density of vacancies, the ground state is infinitely degenerate for $L = \infty$, each configuration of the kind

$$(0)\cdots(0)(+)\cdots(+)(-)\cdots(-)(0)\cdots(0)(+)\cdots(+)(-)\cdots(-)\cdots \tag{2}$$

being a stationary state. At q different from zero and finite L, if one starts with an arbitrary configuration, the system organizes itself into only three blocks

$$(0)(0)\cdots(0)(+)(+)\cdots(+)(-)(-)\cdots(-) \tag{3}$$

which cover the lattice. Translational invariance is respected but the blocks hop from one position to another with a flip time which increases exponentially with L. At the critical point $q = 1$ the flip time increases algebraically with L. This implies that, for $q < 1$, translational invariance is spontaneously broken. Since each of the three blocks contains one kind of particle only, we call it the 'pure' phase. This situation is illustrated by the behaviour of the two-point correlation functions. If we denote by $p(k)$, $m(k)$ and $v(k)$ the concentration of positive particles, negative particles and vacancies, respectively, the unconnected correlation functions are

$$\begin{aligned} &c_{0,0}(k) = \langle v(0)v(k)\rangle \\ &c_{+,-}(k) = \langle p(0)m(k)\rangle \qquad \text{etc.} \end{aligned} \tag{4}$$

In figure 3 we show the typical behaviour of two correlation functions for two small lattice sizes ($L = 75$ and 100). Note that we have chosen $z = k/L$ rather than k as variable. It is straightforward to convince oneself that these correlation functions correspond to the block

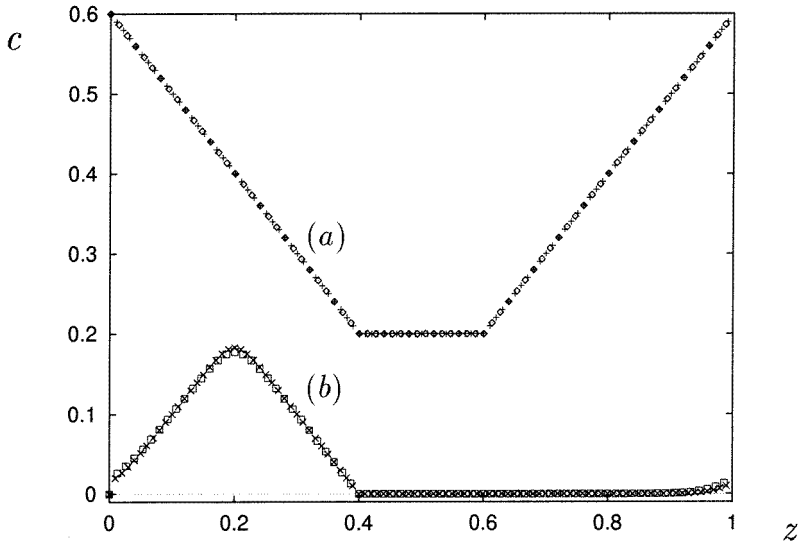

Figure 3. The two-point correlation functions in the 'pure' (broken) phase ($q = 0.5$), with $p = m = 0.2$ and $L = 75(\Diamond, \Box)$ and $100(+, \times)$. (a) gives the $c_{0,0}$ and (b) the $c_{+,-}$ correlation function.

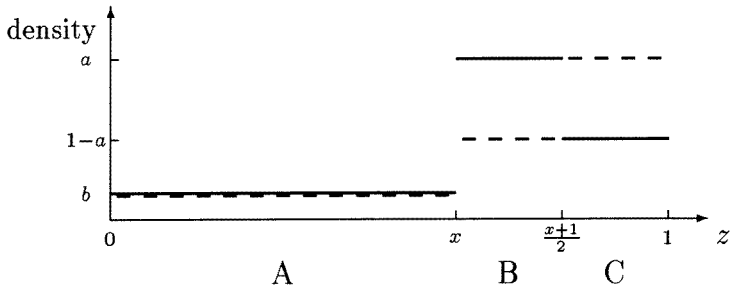

Figure 4. Density profiles in the block picture. The full lines denote positive particles. The dashed lines denote negative particles.

picture (3) described above. The fact that the blocks are already 'pure' for such small lattice sizes reflects the exponential behaviour mentioned before.

If q is slightly greater than 1, the charged particles are still jammed in the charged blocks (the drift is roughly $q - 1$ compared to 1 in the neutral block and positive particles cannot move in a positive block). As a result, we expect the formation of three blocks with a mixed composition as shown in figure 4. Since we are describing macroscopic blocks the appropriate variable is $z = k/L$. For a given average density $p = m$, we have a neutral block A of length x which is no longer pure but contains an equal number of positive and negative particles. Next, we have a positive block B with a density a of positive particles and $1 - a$ of negative particles (no vacancies). Finally, we have a block C, which is the charge-conjugated version of B. These blocks are not pinned. The beginning of A can be found with equal probability in the whole interval $0 \leqslant z < 1$. Charge conservation gives

$$b = \frac{1}{2}\left(1 - \frac{1 - p - m}{x}\right). \tag{5}$$

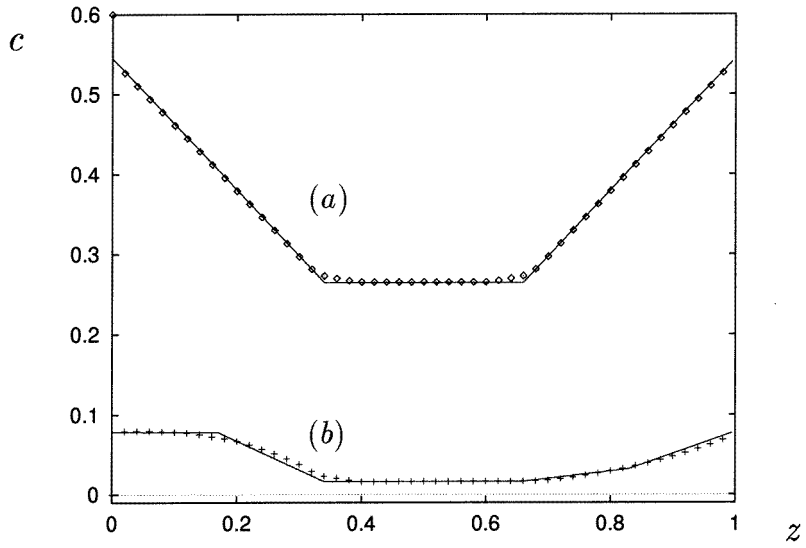

Figure 5. The two-point functions in the 'mixed' phase ($q = 1.2$) for $p = m = 0.2$, $L = 600$. (a) gives the $c_{0,0}$ and (b) the $c_{+,-}$ correlation function. The full lines are given by the block picture. The values of a and x are given in table 1.

Table 1. Estimates of a, b and x for various $1 \leqslant q \leqslant q_c$. The numbers were determined by fitting the data for up to $L = 1000$. The errors are of the order 0.03 for a and 0.01 for x. b is given by equation (5).

$p = m = 0.2$				$p = m = 0.4$			
q	a	b	x	q	a	b	x
1.0	1	0	0.60	1.0	1	0	0.20
1.1	0.71	0.016	0.62	1.2	0.67	0.04	0.22
1.2	0.65	0.045	0.66	1.4	0.66	0.1	0.25
1.3	0.63	0.077	0.71	1.6	0.6	0.17	0.30
1.4	0.62	0.11	0.76	1.8	0.65	0.32	0.55

In this picture, the mobility can be computed from the densities in the A block as

$$\mu = \frac{(q-3)b^2 + b}{p}. \tag{6}$$

This picture cannot be exact since a smooth transition of the charged density profiles should exist between the blocks. This is why we have used the A block and rather than B block to compute the mobility. As q varies between 1 and q_c one expects the A block to take over the whole ring.

In figure 5 we show two correlation functions, obtained from Monte Carlo simulations and from the predictions of the block picture, for $p = m = 0.2$, $q = 1.2$ and $L = 600$. The parameters a and x were obtained by fitting the $c_{+,+}$ correlation function (see table 1), the $c_{0,0}$ and $c_{+,-}$ functions being then determined. We have checked all the correlation functions against the model. The mobility as determined by equation (6) gives $\mu = 0.209$ as compared to the measured value $\mu = 0.210$ ($L = 600$ sites). The consistency of the block picture with the Monte Carlo simulation was checked for various values of q and

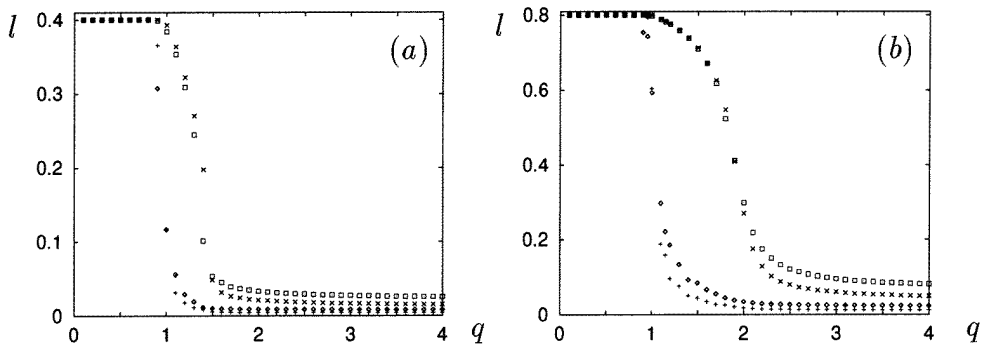

Figure 6. Average length (in units of L) of a string of charged particles ($L = 200(\diamond)$, $400(+)$) and of the longest string of charged particles ($L = 200(\square)$, $400(\times)$) as functions of q. (a) has $p = m = 0.2$, while (b) has $p = m = 0.4$.

densities. Some values for the parameters a, b and x are given in table 1. We have also observed that for $L \geqslant 100$ the block picture is already a good approximation if one uses L-dependent parameters a and x rather than their asymptotic values.

The block picture for the 'mixed' phase can also be checked in another way. We have determined the average length l (in units of L) of a string of charged particles (no vacancies) for various values of q. The data are shown in figure 6 for two densities. One notices that for $q < 1$ the values correspond to the average densities, since we are in the pure phase. For q slightly greater than 1 the average length of the charged strings drops to zero as one expects for the 'mixed' phase. Next one considers a different quantity. For each Monte Carlo configuration one takes the length of the longest string of charged particles and averages over configurations. This quantity should give an independent estimate for x since the average length of the longest string in units of L gives $1 - x$. Comparing the values given in table 1 with the values in figure 6 for the corresponding values of q, one can check that this is indeed the case.

Finally, we have checked that in the disordered phase, $q > q_c$, no macroscopic structures are present. This is in agreement with the exact results obtained for $q = 3$ and 4.

A confirmation of the phase structure described above was obtained by studying the spectrum of the quantum chain Hamiltonian [9] describing the dynamics of the processes given by equation (1). Considering chains of up to 10 sites, we numerically investigated the q dependence of the first energy levels and observed level crossings at values of q compatible with q_c.

A complete description of the results presented here can be found in [7]. There, we also give the phase diagram for the case in which fewer than three of the four rates in equation (1) are taken as equal.

Before closing this letter, we mention that in a two-state model on a ring with six-body dynamics, describing a sedimenting colloidal crystal, Lahiri and Ramaswamy also found interesting macroscopic structures [10].

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Note added in proof. After this work was completed, we have learned that Evans *et al* [11] have reached similar results in a different model.

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